# Vapor-Liquid Equilibria for the Ternary System

## Methyl Acetate–Benzene–Cyclohexane

ISAMU NAGATA<sup>1</sup>

Department of Chemical Engineering, Kyoto University, Kyoto, Japan

**S**EPARATION of the benzene-cyclohexane mixtures resulting from hydrogenation of benzene into pure constituents is industrially important. This is effectively accomplished by azeotropic or extractive distillation of the mixtures with an entrainer. Several investigators have reported vapor-liquid equilibrium data for ternary systems including benzene and cyclohexane, which are needed to design suitable distillation equipment (3, 4, 5, 7, 13, 24, 25).

The objectives of this investigation are: to determine vapor-liquid equilibrium data on the methyl acetatebenzene-cyclohexane system and to correlate the experimental data by two methods; one is an analytical method expressing logarithm of activity coefficients, and the other is an algebraic one, for comparison.

### EXPERIMENTAL

**Purity of Compounds.** First grade (Japanses Industrial Standards) methyl acetate was purified in accordance with the procedure by Hurd and Strong (12). Cyclohexane of guaranteed reagent grade was used without further purification. Special grade (Japanese Industrial Standards) benzene was purified by recrystallization repeated twice. The physical properties of the pure compounds are shown in Table I.

Analytical Method. The binary mixtures were analyzed by measuring refractive indices. The ternary mixtures were analyzed by combining refractive index and density measurements. Refractive index measurements were made using a Shimadzu Pulfrich refractometer. Prism temperature was controlled at  $25 \pm 0.1^{\circ}$  C. by circulating water from a constant temperature bath. Monochromatic light was obtained from a sodium lamp. Modified Lipkin-type pycnometers were used for density determination. The values of refractive index and density were reproducible within  $\pm 0.0001$ . Figure 1 is a ternary calibration diagram constructed from analytical data listed in Table II.

Temperatures were measured by a copper-constant an thermocouple which was calibrated against a standard thermometer and was connected with a Yokogawa P-7 potentiometer. Observed temperatures were corrected to normal boiling points (20) and are believed to be accurate within  $\pm 0.05^{\circ}$  C.

Vapor-liquid equilibrium data on the mixtures were obtained under atmospheric pressure using a Colburn vapor-recirculating still as described by Grisword and Buford (8). Atmospheric pressure recorded during experimental runs showed small deviations from 760 mm. of

<sup>1</sup> Present address, Kanazawa University, Kanazawa, Japan.

mercury (average deviation of  $\pm 3$  mm. Hg.). Such deviations affect observed x-y data negligibly. No correction was made on them.

### EXPERIMENTAL RESULTS

**Binary Systems.** The liquid phase activity coefficients were calculated using the following equation:

$$\gamma_i = \pi \, y_i / x_i p_i \tag{1}$$

The vapor pressure data for methyl acetate were obtained from the compilation of Timmermans (27). The Antoine equations cited by Lange (14) were used to calculate the vapor pressures of benzene and cyclohexane. Binary equilibrium data are shown in Table III.

The area condition for isobaric binary systems has been shown by Herington (11) and Thijssen (26) to be

$$\int_0^1 \log \frac{\gamma_1}{\gamma_2} dx_1 = \frac{1}{2.3} \int_0^1 \frac{\Delta H_m}{RT^2} \left( \frac{dT}{dx_1} \right)_\rho dx_1$$
(2)

For close-boiling point and some azeotropic systems the right side of Equation 2 is nearly zero. The benzene-cyclohexane system is an example of this statement. In order to check thermodynamic consistency for a binary system showing an appreciable difference between both sides of Equation 2 the heat of mixing data at boiling points for the same system are necessary. Such data for the present systems were not available. So no further calculations using Equation 2 were made. Chao (2) modified the Redlich-Kister equations for isobaric conditions, accounting for the requirements of Equation 2. The equation used to correlate the experimental data on log  $\gamma_1/\gamma_2$  in terms of liquid compositions is

$$\log \frac{\gamma_1}{\gamma_2} = a + b(x_2 - x_1) + c(6x_1x_2 - 1)$$
(3)

Constants of the equation are given in Table IV. The activity coefficients of the separate components are expressed by

$$\log \gamma_1 = x_1 x_2 [B + C(x_1 - x_2)] + x_2 [a + b(x_2 - x_1) + c(6x_1 x_2 - 1)]$$
(4)

$$\log \gamma_2 = x_1 x_2 [B + C(x_1 - x_2)] - x_1 [a + b(x_2 - x_1) + c(6x_1 x_2 - 1)]$$
(5)

B and C in Equations 4 and 5 are not independent of b and c respectively. They must fulfill the following relations to satisfy thermodynamic requirements (19).

$$B = b, C = c - a \tag{6}$$

	Iable	I. Properties of	Pure Compour	as		
	Methyl A	Acetate	Ben	zene	Cyc	lohexane
Property	Exptl.	Lit.	Exptl.	Lit. (28)	Exptl.	Lit.
Density, 25° C. Refractive index, 25° C.	0.9273 1.3589	0.9273(28)	$0.8739 \\ 1.4979$	$0.8737 \\ 1.4979$	$0.7738 \\ 1.4236$	$0.7736(3) \\ 1.4235(28)$
Boiling point, ° C., at 760 mm. Hg.	1.3615(20/D) 56.8	1.3614(1) 56.8(1)	80.1	80.1	80.7	80.7(28)

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x,°	x. <sup>b</sup>	$d_{1}^{25}$	$n^{25}$	r,	r	d <sup>25</sup>	$n^{25}_{\rm D}$	Υ,	r	$d_{4}^{25}$	$n^{25}_{D}$
0.923	0.077	0.9218	1 3698	0.761	0	0.8706	1 3730	0.399	0	0.8123	1 3963
0.872	0.128	0.9181	1.3769	0.661	0.131	0.8704	1.3893	0.341	0.145	0.8200	1.4095
0.798	0.202	0.9131	1.3879	0.527	0.308	0.8708	1.4113	0.258	0.353	0.8315	1.4291
0.714	0.286	0.9081	1.3997	0.465	0.388	0.8708	1.4212	0.225	0.435	0.8365	1.4370
0.647	0.353	0.9041	1.4094	0.380	0.500	0.8711	1.4351	0.189	0.525	0.8420	1.4462
0.547	0.453	0.8986	1.4236	0.325	0.573	0.8714	1.4442	-0.163	0.593	0.8461	1.4531
0.448	0.552	0.8920	1.4372	0.262	0.655	0.8718	1.4546	0.125	0.686	0.8522	1.4628
0.343	0.657	0.8881	1.4017	0.180	0.700	0.8721	1.4071	0.088	0.781	0.8083	1.4730
0.298	0.702	0.8814	1.4070	0.120	0.032	0.0121	1.4705	0.000	0.655	0.0000	1.4014
0.100	0.010	0.0014	1.4120	0.678	0	0.8546	1.3782	0.316	0	0.8021	1.4020
0	0.078	0.7791	1.4276	0.576	0.151	0.8569	1.3957	0.258	0.185	0.8132	1.4175
Ó	0.176	0.7860	1.4328	0.457	0.327	0.8602	1.4161	0.220	0.304	0.8208	1.4277
0	0.281	0.7941	1.4389	0.394	0.418	0.8618	1.4272	0.189	0.403	0.8274	1.4368
0	0.379	0.8025	1.4452	0.365	0.462	0.8626	1.4321	0.160	0.495	0.8338	1.4453
0	0.460	0.8100	1.4507	0.275	0.594	0.8652	1.4482	0.086	0.727	0.8512	1.4684
0	0.561	0.8202	1.4583	0.229	0.662	0.8667	1.4565	0.045	0.857	0.8617	1.4823
0	0.638	0.8281	1.4642	0.109	0.840	0.8703	1.4782	0.269	0	0.7069	1 4051
0	0.714	0.8463	1.4707	0.624	0	0.8455	1 3816	0.200	0 175	0.7908	1 4051
õ	0.867	0.8557	1 4847	0.535	0 143	0.8491	1.3975	0.186	0.305	0.8166	1 4298
U	0.001	0.0001	1,1011	0.477	0.237	0.8516	1.4079	0.152	0.431	0.8260	1.4407
0.945	0	0.9129	1.3621	0.400	0.360	0.8549	1.4221	0.128	0.522	0.8326	1.4490
0.787	0.167	0.9041	1.3854	0.333	0.467	0.8580	1.4346	0.103	0.616	0.8402	1.4578
0.679	0.281	0.8990	1.4011	0.266	0.574	0.8608	1.4467	0.082	0.694	0.8464	1.4656
0.600	0.365	0.8954	1.4127	0.193	0.690	0.8641	1.4606	0.058	0.784	0.8541	1.4747
0.501	0.470	0.8911	1.4270	0.081	0.870	0.8696	1.4821	0.042	0.844	0.8593	1.4809
0.411	0.565	0.8874	1,4400	0 550	0	0.0004	1.0005	0.154	0	0 2050	1 4190
0.342	0.637	0.8850	1.4498	0.550	0 169	0.8334	1.3865	0.154	0 171	0.7896	1.4132
0.246	0.737	0.0010	1.4051	0.401	0.162	0.8391	1.4034	0.128	0.171 0.371	0.7973	1 4240
0.175	0.011	0.0751	1.4720	0.362	0.342	0.8461	1.4227	0.082	0.465	0.8201	1.4470
0.863	0	0.8928	1.3669	0.272	0.505	0.8527	1.4406	0.072	0.532	0.8260	1.4525
0.727	0.157	0.8887	1.3879	0.216	0.608	0.8570	1.4522	0.056	0.638	0.8358	1.4620
0.595	0.310	0.8853	1.4080	0.182	0.669	0.8593	1.4591	0.045	0.708	0.8424	1.4683
0.511	0.407	0.8830	1.4209	0.082	0.851	0.8672	1.4803	0.031	0.799	0.8517	1.4773
0.457	0.471	0.8816	1.4292				1 0011	0.022	0.860	0.8580	1.4832
0.362	0.580	0.8799	1.4442	0.478	0	0.8230	1.3911	0.076	0	0.7704	1 4105
0.300	0.652	0.8788	1.4531	0.402	0.158	0.8299	1.4066	0.076	0 180	0.7794	1,4180
0.208	0.758	0.0771	1,4070	0.349	0.269	0.6301	1,4179	0.003	0.160	0.7919	1.4255
0.130	0.045	0.0101	1.4775	0.300	0.374	0.8454	1 4395	0.043	0.305	0.8129	1 4471
0.820	0	0.8831	1 3694	0.208	0.565	0.8498	1.4488	0.035	0.548	0.8228	1.4478
0.744	0.093	0.8818	1.3814	0.132	0.723	0.8584	1.4662	0.028	0.640	0.8321	1.4631
0.595	0.274	0.8793	1.4048	0.076	0.841	0.8649	1.4795	0.021	0.719	0.8403	1.4702
0.500	0.390	0.8780	1.4198					0.015	0.801	0.8494	1.4777
0.443	0.460	0.8772	1.4286					0.010	0.863	0.8568	1.4839
0.344	0.581	0.8763	1.4442								
0.274	0.660	0.8758	1.4551								
0.198	0.759	0.8748	1.4070	• M	ole fraction	n of methyl	acetate <sup>b</sup> M	ole fraction	of henzene		
0.152	0.815	0.8740	1,4/43	141	ione maction	n or memyra	acciaic. IVI	ore macuph	or benzelle	•	

Table III. Binary Vapor-Liquid Equilibrium Data

	Methy	l Acetate-Cy	clohexane			Be	enzene-Cycloł	nexane	
t, ° C.	$x_1^a$	ν,	$\gamma_1$	γ <sub>2</sub>	$t, \circ C.$	$x_1^a$	y <sub>1</sub>	$\gamma_1$	<b>γ</b> <sub>2</sub>
74.3	0.033	0.182	3.130	1.031	79.5	0.101	0.131	1.321	1.003
68.4	0.085	0.350	2.804	1.046	78.9	0.171	0.211	1.278	1.005
64.9	0.142	0.443	2.383	1.073	78.4	0.256	0.293	1.206	1.020
59.7	0.283	0.575	1.843	1.170	77.8	0.343	0.376	1.177	1.038
59.0	0.313	0.594	1.761	1.195	77.5	0.428	0.445	1.127	1.071
57.9	0.373	0.625	1.614	1.288	77.4	0.525	0.529	1.088	1.098
56.8	0.478	0.664	1.389	1.406	77.4	0.571	0.564	1.074	1.122
56.7	0.507	0.673	1.333	1.454	77.6	0.665	0.645	1.048	1.164
56.0	0.616	0.714	1.194	1.674	77.9	0.759	0.728	1.027	1.230
55.8	0.688	0.744	1.120	1.857	78.2	0.810	0.777	1.017	1.268
55.7	0.722	0.759	1.093	1.969	78.6	0.863	0.834	1.012	1.297
55.5	0.781	0.789	1.054	2.204	79.3	0.945	0.926	1.004	1.405
55.55	0.835	0.820	1.027	2.491					
55.8	0.940	0.914	1.007	3.244					
" Mole f	raction of me	ethyl acetate.			<sup>a</sup> Mole	e fraction of b	enzene.		

## Table IV. Constants Used in Expressing Log $\gamma_i/\gamma_j$

System	$a_{ii}$	$\boldsymbol{b}_{ij}$	$c_{ij}$
Methyl Acetate-Benzene	-0.0308	0.1245	-0.0183
Benzene-Cyclohexane	0	0.1447	0.0121
Cyclohexane-Methyl Acetate	0.0257	0.5274	0.0016

Experimental and calculated activity coefficients are plotted in Figure 2. It is practical to compare calculated vapor conpositions and boiling points with the experimental data to estimate the accuracy of numerical correlations (Figures 3 and 4). The calculated vapor compositions closely agree with the observed values, but the calculated temperatures are higher than the experimental results. The





Figure 3. Log  $\gamma$  vs. composition diagram

- 1 Benzene-cyclohexone
- 2 Methyl acetate-benzene
- 3 Methyl acetate-cyclahexane
- Experimental
- Calculated

vapor-liquid equilibrium data for the benzene-cyclohexane system have been reported by several investigators (3, 6, 20, 22, 23, 24). The author's results agreee well with those of Donald and Ridgway (6) and of Sieg (23). The data on the methyl acetate-benzene system were given by the author (17). The data on the methyl acetate-cyclohexane system are not reported in the literature. This system shows azeotropic behavior.

Ternary system. The ternary vapor-liquid equilibrium data are listed with the calculated results in Table V. The logarithm of activity coefficients of the ternary system are expressed by an extension of the binary equations (17).

$$[b_{12} + c_{12}(x_1 - x_2) + \dots] - 2x_2 x_3 [b_{23} + c_{23}(x_2 - x_3) + \dots]$$
  
+  $[(x_3 - x_1)(x_2 + x_3) + x_1 x_2] [b_{31} + c_{31}(x_3 - x_1) + \dots]$   
+  $[x_1 x_2(x_2 + x_3) + x_1 x_2^2] (c_{12} + \dots) + [x_2 x_3(x_2 + x_3) - 2x_2^2 x_3]$   
 $(c_{23} \dots) + [-2x_3 x_1(x_2 + x_3) + x_1 x_2 x_3] (c_{31} + \dots)$   
+  $[(x_2 + x_3)(x_2 x_3 - x_1 x_2) - x_2(x_1 x_3 - x_1 x_2)] [b + c_1(x_2 - x_3)$   
+  $c_2(x_3 - x_1) + c_3(x_1 - x_2) + \dots] + x_1 x_2 x_3[(x_2 + x_3) - (x_2 - x_3)]$   
 $(c_1 - 2c_2 + c_3) - x_2(2c_1 - c_2 - c_3) + \dots]$ (7)

Cyclic advancement of the subscripts gives  $\log \gamma_2$  and  $\log \gamma_3$ .  $G^E$  values are expressed by

$$G_{123}^{E} = G_{12}^{E} + G_{23}^{E} + G_{31}^{E} + 2.3 RT x_1 x_2 x_3$$

 $\log \gamma_1 = (G_{123}^E/2.3RT) + a_{12}x_2 - a_{32}x_3 + [x_2(x_2 + x_3) - x_1x_2]$ 

$$[B + C_1(x_2 - x_3) + C_2(x_3 - x_1) + C_3(x_1 - x_2) + \dots]$$
(8)

$$G_{12}^{E} = 2.3 RT x_1 x_2 [B_{12} + C_{12}(x_1 - x_2) + \dots]$$
(9)

In the numerical determination of ternary constants from the data, b was assumed to be equal to  $\dot{B}$ , and the other ternary constants were set to zero, for the sake of simplicity. The value of b = 0.102 was determined in the concentration range where the ternary constant b has a significant contribution to  $\log \gamma$  and was retained to give the precise fit of the experimental and the calculated data as shown in Figure V. The deviation of the calculated vapor compositions from the experimental results is 0.007 for methyl acetate, 0.007 for benzene, and 0.005 for cyclohexane, respectively. The calculated temperatures average 0.5° C. higher than the observed data. Repeated trail calculations indicated that the benzene-cyclohexane azeotrope diminishes completely at the liquid concentration of methyl acetate of 0.293 mole fraction. There is no ternary azeotrope.

Table V. Ternary Vapor-Liquid Equilibrium Data tor the Methyl Acetate-Benzene-Cyclohexane

	Coeff."	$\gamma^3$ 1.436 1.602 1.607 1.565 1.738	$\begin{array}{c} 1.882 \\ 2.084 \\ 2.198 \\ 2.484 \\ 2.826 \end{array}$	2.662 1.368 1.387 1.499 1.615	$\begin{array}{c} 1.494 \\ 1.876 \\ 2.072 \\ 2.340 \\ 1.287 \end{array}$	1.331 1.392 1.474 1.672 1.754	$\begin{array}{c} 1.948\\ 2.156\\ 1.230\\ 1.280\\ 1.303\end{array}$	$\begin{array}{c} 1.479\\ 1.528\\ 1.653\\ 1.859\\ 1.172\end{array}$	1.153 1.205 1.256 1.331 1.416
	Activity	$\gamma^{2}$ 1.002 1.022 1.021 1.013 1.046	$\begin{array}{c} 1.076\\ 1.119\\ 1.143\\ 1.199\\ 1.258\end{array}$	$\begin{array}{c} 1.233\\ 1.012\\ 1.008\\ 1.017\\ 1.037\end{array}$	$\begin{array}{c} 1.065\\ 1.085\\ 1.161\\ 1.184\\ 1.012\end{array}$	$\begin{array}{c} 1.014 \\ 1.020 \\ 1.035 \\ 1.063 \\ 1.085 \end{array}$	$\begin{array}{c} 1.118\\ 1.161\\ 1.025\\ 1.022\\ 1.026\\ 1.026\end{array}$	$\begin{array}{c} 1.041 \\ 1.053 \\ 1.081 \\ 1.119 \\ 1.040 \end{array}$	$\begin{array}{c} 1.047\\ 1.038\\ 1.042\\ 1.042\\ 1.044\\ 1.038\end{array}$
	Calcd.	$\gamma_1$ 1.279 1.162 1.162 1.159 1.184 1.116	1.095 1.055 1.038 1.019 1.007	1.013 1.330 1.274 1.226 1.165	$\begin{array}{c} 1.102\\ 1.098\\ 1.041\\ 1.020\\ 1.360\end{array}$	$\begin{array}{c} 1.203\\ 1.296\\ 1.235\\ 1.147\\ 1.124\end{array}$	$\begin{array}{c} 1.082\\ 1.055\\ 1.500\\ 1.435\\ 1.386\end{array}$	$\begin{array}{c} 1.236\\ 1.211\\ 1.211\\ 1.162\\ 1.106\\ 1.620\end{array}$	1.676 1.565 1.448 1.360 1.282
	Coeff.	$\gamma^{3}$ 1.385 1.560 1.663 2.121 1.852	$\begin{array}{c} 1.996\\ 2.059\\ 2.645\\ 2.765\\ 2.962\end{array}$	2.743 1.369 1.378 1.378 1.458 1.458	$\begin{array}{c} 1.778 \\ 1.856 \\ 2.255 \\ 2.427 \\ 1.295 \end{array}$	$\begin{array}{c} 1.367\\ 1.450\\ 1.506\\ 1.683\\ 1.696\end{array}$	$\begin{array}{c} 1.914 \\ 2.139 \\ 1.260 \\ 1.303 \\ 1.331 \end{array}$	$\begin{array}{c} 1.507\\ 1.558\\ 1.558\\ 1.661\\ 1.886\\ 1.212\end{array}$	$\begin{array}{c} 1.220\\ 1.235\\ 1.285\\ 1.365\\ 1.443\end{array}$
	Activity	$\gamma^{2}$ 1.038 1.063 1.063 1.037 1.028 1.047	$\begin{array}{c} 1.072 \\ 1.114 \\ 1.044 \\ 1.153 \\ 1.255 \end{array}$	1.223 1.027 1.039 1.045 1.057	$\begin{array}{c} 1.086\\ 1.096\\ 1.130\\ 1.185\\ 1.039\end{array}$	0.987 1.047 1.062 1.064 1.211	$\begin{array}{c} 1.174 \\ 1.184 \\ 1.043 \\ 1.058 \\ 1.056 \end{array}$	$\begin{array}{c} 1.057\\ 1.068\\ 1.111\\ 1.066\\ 1.055\end{array}$	1.031 1.065 1.076 1.068 1.098
	Obsd.	$\gamma_1$ 1.277 1.168 1.168 1.216 1.216 1.185 1.171	$\begin{array}{c} 1.136\\ 1.075\\ 1.040\\ 1.015\\ 0.994\end{array}$	$\begin{array}{c} 1.014\\ 1.253\\ 1.317\\ 1.199\\ 1.186\end{array}$	$\begin{array}{c} 1.107\\ 1.128\\ 1.114\\ 1.027\\ 1.386\end{array}$	$\begin{array}{c} 1.333\\ 1.242\\ 1.228\\ 1.184\\ 1.110\end{array}$	$\begin{array}{c} 1.116\\ 1.064\\ 1.474\\ 1.385\\ 1.392\end{array}$	$\begin{array}{c} 1.242 \\ 1.251 \\ 1.184 \\ 1.134 \\ 1.421 \end{array}$	$\begin{array}{c} 1.554 \\ 1.522 \\ 1.438 \\ 1.398 \\ 1.295 \end{array}$
<i>с</i> і	lcd.	/ 75.1 68.5 69.8 71.3 66.4	64.0 63.0 61.3 59.8 58.8	58.6 73.6 71.0 68.7 66.0	64.1 62.5 59.6 58.2 73.1	70.2 67.5 65.4 63.1 61.3	59.9 57.5 72.1 68.6 67.6	63.9 62.4 59.9 72.9	74.3 69.3 65.6 63.6 61.1
emp., ° (	Ca	a 75.9 69.3 70.4 72.2 66.7	64.0 62.5 60.9 59.1 58.2	57.7 74.3 72.4 69.1 66.1	64.8 62.0 59.1 57.8 74.0	72.1 68.0 65.4 62.8 60.6	59.0 57.0 72.7 69.5 67.9	63.8 62.1 59.5 57.4 73.6	74.8 69.7 65.6 63.7 61.1
L		Obsd. 75.0 68.9 69.5 71.3 65.8	$\begin{array}{c} 63.2 \\ 62.2 \\ 60.8 \\ 59.1 \\ 58.3 \end{array}$	57.7 74.3 71.6 69.0 65.5	64.0 61.5 58.3 57.5 73.4	70.7 67.9 65.1 62.2 60.5	58.3 56.8 72.3 69.3 67.4	63.5 61.4 59.0 56.8 73.7	74.7 69.4 65.2 62.9 60.6
		$y_3$ 0.078 0.097 0.064 0.057 0.096	$\begin{array}{c} 0.104 \\ 0.077 \\ 0.103 \\ 0.092 \\ 0.060 \end{array}$	$\begin{array}{c} 0.094 \\ 0.153 \\ 0.169 \\ 0.167 \\ 0.157 \\ 0.153 \end{array}$	$\begin{array}{c} 0.130\\ 0.155\\ 0.144\\ 0.156\\ 0.229\end{array}$	$\begin{array}{c} 0.223\\ 0.221\\ 0.217\\ 0.194\\ 0.209\end{array}$	$\begin{array}{c} 0.195\\ 0.197\\ 0.286\\ 0.279\\ 0.269\end{array}$	0.237 0.243 0.249 0.249 0.241 0.343	0.366 0.324 0.295 0.295 0.291
	Calcd. <sup>6</sup>	$egin{array}{c} y_2 \\ 0.771 \\ 0.510 \\ 0.569 \\ 0.638 \\ 0.423 \end{array}$	$\begin{array}{c} 0.333\\ 0.293\\ 0.213\\ 0.147\\ 0.102\end{array}$	$\begin{array}{c} 0.090\\ 0.659\\ 0.568\\ 0.468\\ 0.373\end{array}$	$\begin{array}{c} 0.322 \\ 0.245 \\ 0.132 \\ 0.082 \\ 0.578 \end{array}$	$\begin{array}{c} 0.485\\ 0.389\\ 0.314\\ 0.250\\ 0.175\end{array}$	$\begin{array}{c} 0.129\\ 0.057\\ 0.498\\ 0.402\\ 0.354\end{array}$	$\begin{array}{c} 0.255\\ 0.203\\ 0.122\\ 0.058\\ 0.480\end{array}$	0.507 0.369 0.256 0.212 0.139
		$y_1$ 0.151 0.393 0.367 0.367 0.305 0.481	0.563 0.630 0.684 0.684 0.761 0.838	0.816 0.188 0.263 0.375 0.474	$\begin{array}{c} 0.548 \\ 0.600 \\ 0.724 \\ 0.762 \\ 0.193 \end{array}$	$\begin{array}{c} 0.292 \\ 0.390 \\ 0.469 \\ 0.556 \\ 0.616 \end{array}$	0.676 0.746 0.216 0.319 0.377	$\begin{array}{c} 0.508 \\ 0.554 \\ 0.629 \\ 0.629 \\ 0.771 \\ 0.177 \end{array}$	0.127 0.307 0.449 0.493 0.570
.udi		$egin{array}{c} y_{\scriptscriptstyle 3} \\ 0.073 \\ 0.064 \\ 0.052 \\ 0.087 \end{array}$	$\begin{array}{c} 0.094 \\ 0.077 \\ 0.095 \\ 0.087 \\ 0.057 \end{array}$	$\begin{array}{c} 0.092\\ 0.144\\ 0.171\\ 0.148\\ 0.148\\ 0.142\end{array}$	$\begin{array}{c} 0.102 \\ 0.146 \\ 0.129 \\ 0.152 \\ 0.223 \end{array}$	$\begin{array}{c} 0.222\\ 0.214\\ 0.208\\ 0.186\\ 0.202\end{array}$	$\begin{array}{c} 0.188\\ 0.191\\ 0.282\\ 0.265\\ 0.264\end{array}$	$\begin{array}{c} 0.230\\ 0.236\\ 0.242\\ 0.235\\ 0.338\\ 0.338\end{array}$	$\begin{array}{c} 0.360 \\ 0.320 \\ 0.312 \\ 0.292 \\ 0.288 \end{array}$
apor Com	Calcd."	$egin{array}{c} y_2 \\ 0.768 \\ 0.497 \\ 0.550 \\ 0.622 \\ 0.405 \end{array}$	$\begin{array}{c} 0.311 \\ 0.262 \\ 0.196 \\ 0.133 \\ 0.092 \end{array}$	$\begin{array}{c} 0.080\\ 0.666\\ 0.564\\ 0.462\\ 0.360\end{array}$	$\begin{array}{c} 0.312 \\ 0.228 \\ 0.120 \\ 0.073 \\ 0.594 \end{array}$	$\begin{array}{c} 0.507 \\ 0.385 \\ 0.305 \\ 0.236 \\ 0.236 \\ 0.160 \end{array}$	$\begin{array}{c} 0.115\\ 0.049\\ 0.502\\ 0.405\\ 0.351\end{array}$	$\begin{array}{c} 0.242 \\ 0.190 \\ 0.108 \\ 0.049 \\ 0.492 \end{array}$	0.520 0.370 0.239 0.199 0.122
Ň		$egin{array}{c} y_1 \\ 0.159 \\ 0.415 \\ 0.386 \\ 0.326 \\ 0.508 \end{array}$	0.595 0.661 0.709 0.851	$\begin{array}{c} 0.828\\ 0.190\\ 0.265\\ 0.390\\ 0.498\end{array}$	$\begin{array}{c} 0.586\\ 0.586\\ 0.626\\ 0.751\\ 0.775\\ 0.183\end{array}$	$\begin{array}{c} 0.271 \\ 0.401 \\ 0.487 \\ 0.579 \\ 0.638 \end{array}$	0.697 0.760 0.216 0.330 0.385	$\begin{array}{c} 0.528\\ 0.574\\ 0.650\\ 0.716\\ 0.170\end{array}$	$\begin{array}{c} 0.120\\ 0.310\\ 0.449\\ 0.509\\ 0.590\end{array}$
		$y_3$ 0.073 0.084 0.064 0.068 0.090	$\begin{array}{c} 0.097\\ 0.075\\ 0.114\\ 0.116\end{array}$	$\begin{array}{c} 0.092\\ 0.145\\ 0.166\\ 0.144\\ 0.144\\ 0.143\end{array}$	$\begin{array}{c} 0.118\\ 0.142\\ 0.137\\ 0.156\\ 0.220\\ \end{array}$	$\begin{array}{c} 0.218\\ 0.222\\ 0.211\\ 0.183\\ 0.195\end{array}$	$\begin{array}{c} 0.180\\ 0.189\\ 0.285\\ 0.268\\ 0.265\\ 0.265\end{array}$	$\begin{array}{c} 0.231 \\ 0.235 \\ 0.240 \\ 0.234 \\ 0.250 \end{array}$	$\begin{array}{c} 0.379\\ 0.325\\ 0.315\\ 0.315\\ 0.291\\ 0.288\end{array}$
	Obsd.	$egin{array}{c} y_2 \\ 0.773 \\ 0.503 \\ 0.542 \\ 0.614 \\ 0.392 \end{array}$	$\begin{array}{c} 0.302 \\ 0.258 \\ 0.178 \\ 0.128 \\ 0.092 \end{array}$	$\begin{array}{c} 0.079\\ 0.676\\ 0.567\\ 0.475\\ 0.360\end{array}$	$\begin{array}{c} 0.309\\ 0.226\\ 0.114\\ 0.072\\ 0.597\end{array}$	$\begin{array}{c} 0.494 \\ 0.394 \\ 0.310 \\ 0.231 \\ 0.177 \end{array}$	$\begin{array}{c} 0.118\\ 0.050\\ 0.505\\ 0.416\\ 0.355\end{array}$	$\begin{array}{c} 0.244 \\ 0.186 \\ 0.109 \\ 0.046 \\ 0.500 \end{array}$	$\begin{array}{c} 0.510 \\ 0.375 \\ 0.244 \\ 0.198 \\ 0.127 \end{array}$
		$y_1$ 0.154 0.413 0.394 0.318 0.518	$\begin{array}{c} 0.601 \\ 0.667 \\ 0.708 \\ 0.776 \\ 0.848 \end{array}$	$\begin{array}{c} 0.829\\ 0.179\\ 0.267\\ 0.381\\ 0.497\end{array}$	$\begin{array}{c} 0.573 \\ 0.632 \\ 0.749 \\ 0.772 \\ 0.183 \end{array}$	$\begin{array}{c} 0.288\\ 0.384\\ 0.479\\ 0.586\\ 0.628\end{array}$	$\begin{array}{c} 0.702 \\ 0.761 \\ 0.210 \\ 0.316 \\ 0.380 \end{array}$	0.525 0.579 0.651 0.720 0.720	$\begin{array}{c} 0.111\\ 0.300\\ 0.441\\ 0.511\\ 0.585\end{array}$
	pn.	$\begin{array}{c} x_3 \\ 0.059 \\ 0.078 \\ 0.055 \\ 0.043 \\ 0.078 \end{array}$	$\begin{array}{c} 0.085\\ 0.066\\ 0.082\\ 0.070\\ 0.042\end{array}$	$\begin{array}{c} 0.071\\ 0.129\\ 0.160\\ 0.142\\ 0.142\\ 0.140\end{array}$	$\begin{array}{c} 0.113\\ 0.142\\ 0.126\\ 0.137\\ 0.213\end{array}$	$\begin{array}{c} 0.218\\ 0.229\\ 0.230\\ 0.197\\ 0.221\\ \end{array}$	$\begin{array}{c} 0.195\\ 0.193\\ 0.294\\ 0.294\\ 0.302\end{array}$	$\begin{array}{c} 0.266\\ 0.281\\ 0.292\\ 0.271\\ 0.359\end{array}$	$\begin{array}{c} 0.374 \\ 0.375 \\ 0.401 \\ 0.377 \\ 0.382 \end{array}$
	quid Com	$x_{2}^{u}$ 0.874 0.685 0.732 0.732 0.790 0.594	$\begin{array}{c} 0.488\\ 0.416\\ 0.322\\ 0.222\\ 0.151\end{array}$	$\begin{array}{c} 0.136\\ 0.790\\ 0.715\\ 0.646\\ 0.546\\ 0.546\end{array}$	$\begin{array}{c} 0.480\\ 0.379\\ 0.206\\ 0.129\\ 0.710\end{array}$	$\begin{array}{c} 0.645\\ 0.557\\ 0.474\\ 0.390\\ 0.279\end{array}$	$\begin{array}{c} 0.207 \\ 0.092 \\ 0.620 \\ 0.555 \\ 0.506 \end{array}$	$\begin{array}{c} 0.396\\ 0.322\\ 0.197\\ 0.094\\ 0.580\end{array}$	0.586 0.495 0.367 0.325 0.220
	Lic	$x_1^{i}$ 0.067 0.237 0.213 0.213 0.167 0.328	$\begin{array}{c} 0.427\\ 0.518\\ 0.596\\ 0.708\\ 0.807\end{array}$	$\begin{array}{c} 0.793 \\ 0.081 \\ 0.125 \\ 0.212 \\ 0.314 \end{array}$	$\begin{array}{c} 0.407\\ 0.479\\ 0.668\\ 0.734\\ 0.077\end{array}$	$\begin{array}{c} 0.137\\ 0.214\\ 0.296\\ 0.413\\ 0.500\end{array}$	$\begin{array}{c} 0.598 \\ 0.715 \\ 0.086 \\ 0.151 \\ 0.192 \end{array}$	0.338 0.397 0.511 0.635 0.061	$\begin{array}{c} 0.040\\ 0.130\\ 0.232\\ 0.298\\ 0.398\end{array}$

$\begin{array}{c} 1.063\\ 1.106\\ 1.177\\ 1.077\\ 1.077\\ 1.047\\ 1.029\\ 1.029\\ 1.034\\ 3.087\\ 3.087\end{array}$	$\begin{array}{c} 1.099\\ 1.077\\ 1.077\\ 1.085\\ 1.146\\ 1.146\\ 1.164\\ 1.150\\ 1.150\\ 1.232\\ 1.232\end{array}$	2.055 1.762 1.603 1.603 1.584 2.163 2.163 2.034 1.813 2.203 1.813 2.203 1.003	$\begin{array}{c} 1.092\\ 1.144\\ 1.210\\ 1.146\\ 1.057\\ 1.067\\ 1.092\\ 1.048\\ 1.048\\ 3.399\end{array}$	$\begin{array}{c} 1.130\\ 1.102\\ 1.124\\ 1.262\\ 1.124\\ 1.124\\ 1.173\\ 1.173\\ 1.274\\ 1.177\\ 1.219\\ 1.219\end{array}$	$\begin{array}{c} 1.813\\ 1.745\\ 1.654\\ 1.654\\ 1.959\\ 2.016\\ 1.954\\ 2.099\\ 2.199\\ 0.998\\ 0.998\end{array}$	72.6 67.3 63.1 63.1 61.0 61.0 61.0 63.1 71.1 71.1 71.1 57.6	73.3 68.7 63.5 63.5 63.5 73.9 69.1 63.1 68.6 56.7	73.3 68.2 62.7 60.9 74.0 67.9 67.9 68.6 56.6 56.6 56.6 58.0 58.0 58.0 58.0 56.6	0.490 0.412 0.374 0.380 0.558 0.497 0.558 0.497 0.497 0.497 0.562 0.562 0.562 0.562 0.563 0.063	0.330 0.248 0.139 0.078 0.078 0.278 0.278 0.278 0.203 0.097 0.097 0.097 0.093 0.097 0.033 0.097	0.180 0.340 0.487 0.542 0.164 0.345 0.345 0.351 0.351 0.304	0.485 0.414 0.373 0.359 0.559 0.559 0.574 0.574 0.553 0.062 0.553	0.332 0.241 0.126 0.069 0.069 0.155 0.069 0.155 0.078 0.155 0.078 0.155 0.078 0.078 0.086 0.086	0.183 0.345 0.501 0.577 0.577 0.577 0.577 0.577 0.571 0.493 0.231 0.218 0.218 0.218 0.218 0.218 0.218	<ul> <li>0.498</li> <li>0.421</li> <li>0.373</li> <li>0.355</li> <li>0.567</li> <li>0.568</li> <li>0.572</li> <li>0.558</li> <li>0.068</li> <li>0.068</li> <li>d Mole</li> <li>d Mole</li> </ul>	0.241 0.243 0.128 0.076 0.076 0.286 0.286 0.286 0.080 0.080 0.080 0.088 0.080 0.088 0.020	$\begin{array}{c} 0.161\\ 0.336\\ 0.499\\ 0.569\\ 0.147\\ 0.147\\ 0.355\\ 0.484\\ 0.218\\ 0.354\\ 0.218\\ 0.354\\ 0.912\\ 0.912\\ \end{array}$	0.574 0.545 0.549 0.589 0.589 0.589 0.589 0.589 0.589 0.589 0.589 0.583 0.714 0.783 0.713 0.044 *********************************	0.374 0.374 0.323 0.201 0.113 0.113 0.297 0.113 0.120 0.297 0.120 0.297 0.120 0.297 0.120 0.036 0.036 0.036 0.036	0.132 0.132 0.132 0.250 0.043 0.043 0.043 0.043 0.043 0.043 0.043 0.043 0.005 0.005 0.109 0.920
1.063	1.099	2.055	1.092	1.130	1.813	72.6	73.3	73.3	0.490	0.330	0.180	0.485	0.332	0.183	0.498	0.341	0.161	0.574	0.374	0.052
1.403	1.084	1.333	1.421	1.105	1.341	58.6 67 4	58.5	58.3 20.0	0.324	0.053	0.623	0.319	0.043	0.638	0.320	0.044	0.636	0.467	0.082	0.451
1.167 1.213	1.065 1.067	1.616 1.524	$1.176 \\ 1.255$	$1.114 \\ 1.125$	1.589 1.555	65.5 62.6	66.0 62.9	65.8 62.0	0.369 0.355	$0.212 \\ 0.138$	0.419 0.507	0.369 0.353	$0.202 \\ 0.124$	0.429 0.523	$0.370 \\ 0.355$	$0.210 \\ 0.127$	0.420 0.518	0.505 0.516	$0.299 \\ 0.204$	$0.196 \\ 0.280$
1.147 1.115	1.062 1.073	1.674 1.789	1.173 1.275	1.101 1.234	1.663 1.971	68.9 71.2	69.2 72.2	68.7 71.2	0.387 0.418	0.310 0.362	0.303 0.220	$0.379 \\ 0.417$	0.300 0.367	0.321 0.216	0.381 0.420	0.306 0.370	0.313 0.210	0.473 0.488	0.400 0.439	0.127 0.073
1.121	1.067	1.763	1.158	1.119	1.663	72.7	73.4	72.5	0.407	0.414	0.179	0.406	0.422	0.172	0.412	0.430	0.158	0.454	0.489	0.057
$1.110 \\ 1.146$	1.073 1.063	1.806 1.676	1.172 1.180	111.1	1.834 1.673	72.6 68.7	73.6 69.3	72.5 68.7	0.402 0.375	0.420 0.305	$0.178 \\ 0.316$	0.422 0.380	0.413 0.303	0.165 0.317	0.426 0.383	0.412 0.307	0.162 0.310	0.475 0.473	0.472 0.402	0.053 0.125
1.547	1.081	1.240	1.567	1.120	1.252	58.8	58.2	57.8	0.285	0.075	0.640	0.275	0.062	0.663	0.275	0.064	0.661	0.370	0.120	0.510



Algebraic Method. The algebraic equations enable us to carry out rapid calculation of vapor-liquid equilibrium data. The ordinary three-constant Prahl equation involves more additional terms which improves the fit of the experimental x-y data (9, 10, 15, 21).

The following equation was used to correlated the binary x-y data.

$$\frac{y_i}{y_i} = \frac{x_i}{x_i} \left( \frac{x_i + x_i a_{ij} + x_i x_j d_{ij}}{x_i b_{ii} + x_i c_{ij}} \right)$$
(10)

This equation has another advantage: the ternary (or multicomponent) vapor-liquid equilibrium are predicable from component binary data as suggested by Lu, Li, and Ting (15) if b values satisfy the condition of Equation 11 approximately.

$$b_{12} \cdot b_{23} \cdot b_{31} = 1 \tag{11}$$

A three-index equation based on Equation 10 is expressed by

$$y_1: y_2: y_3 =$$

٩

$$\left(\frac{b_{31}}{b_{12}}\right)^{1/3} x_1(x_1 + x_2a_{12} + x_3 - \frac{c_{31}}{b_{31}} + x_1x_2d_{12} + x_2x_3\alpha_1)$$
  

$$: \left(\frac{b_{12}}{b_{23}}\right)^{1/3} x_2(x_2 + x_3a_{23} + x_1 - \frac{c_{12}}{b_{12}} + x_2x_3d_{23} + x_3x_1\alpha_2)$$
  

$$: \left(\frac{b_{23}}{b_{31}}\right)^{1/3} x_3(x_3 + x_1a_{31} + x_2 - \frac{c_{23}}{b_{23}} + x_3x_1d_{31} + x_1x_2\alpha_3) \quad (12)$$

where  $\alpha$ 's are ternary constants to be determined from the experimental data.

Boiling temperatures of the binary and ternary systems were calculated in accordance with the method proposed by the author (16, 18). For the present ternary mixtures

$$T_{m} = \sum T_{i} y_{ii} + \sum_{i=1}^{n} T_{ii} (y_{ii} + y_{ji}) + T_{123} (y_{123} + y_{231} + y_{312})$$
(13)

where  $y_{123}$  is defined by

$$y_{123} = y_1 \frac{x_2 x_3 \alpha_1}{x_1 + x_2 \alpha_{12} + x_3 \alpha_{31} / b_{31} + x_1 x_2 d_{12} + x_2 x_3 \alpha_1}$$
(14)

Similarly for  $y_{231}$  and  $y_{312}$ . Constants of Equations 12 and



Figure 5. Boiling points vs. composition diagram

- 1 Benzene-cyclohexane
- 2 Benzene-methyl acetate
- 3 Methyl acetate-cyclohexane
- Experimental
- Calculated (Modified Redlich-Kister)
- ---- Calculated (Nagata method)

13 are listed in Table VI. The average deviations of the calculated from the experimental results are as follows:

Vapor Composition

0.015 mole fraction
0.012 mole fraction
0.007 mole fraction
0.5° C.

#### Table VI. Constants of Ternary Algebraic Equations

$a_{12} \\ 0.760$	$b_{12} \\ 0.277$	$c_{12} \\ 0.629$	${d_{12} \atop 0}$	$\alpha_1$ 0.724	$T_{12} \\ 338.7$	$b_{12} \cdot b_{23} \cdot b_{31} \\ 0.876$
$a_{ m 23} \ 1.473$	$b_{23} \\ 1.015$	$c_{23}$ 1.406	$d_{23} = 0$	$\alpha_{2}$ 2.311	$T_{23}$ 348.8	${T}_{123}$ 360.0
$a_{ m 31} 5.009$	$b_{31} \\ 3.115$	$c_{31} \\ 6.698$	$d_{31} - 2.385$	$0^{\alpha_3}$	$T_{13} \\ 327.7$	

## CONCLUSIONS

The methyl acetate is suitable as an entrainer for azeotropic distillation of the benzene-cyclohexane mixtures. Both the analytical and algebraic methods give the boiling temperatures of the same accuracy. The former provides us with more precise vapor compositions than the latter does, although the method is slow.

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### NOMENCLATURE

- $a_{12}, b_{12}, c_{12}; a_{23}, b_{23}, c_{23}; a_{31}, b_{31}, c_{31} = \text{binary constants of modified}$ Redlich-Kister equations
- b,  $c_1$ ,  $c_2$ ,  $c_3$ ; B,  $C_1$ ,  $C_2$ ,  $C_3$  = ternary constants of modified **Redlich-Kister** equations

 $a_{23}, b_{12}, c_{12}, d_{12}; a_{23}, b_{23}, c_{23}, d_{23}; a_{31}, b_{31}, c_{31}, d_{31} = \text{binary constants}$ of algebraic equations

- $d_4^{25}$  = density at 25° C. with respect to water at 4° C.  $G^{\varepsilon}$  = excess free energy
- $\Delta H_m = \text{heat of mixing}$
- $n_{\rm D}^{25}$  = refractive index for sodium D line at 25° C.
- vapor pressure of component i $p_1$ =
- R =gas constant
- T =boiling temperature, ° K.
- $T_{12}, T_{23}, T_{13}, T_{123} =$ constants
  - x = mole fraction in liquid phase
  - y = mole fraction in vapor phase

 $y_{11}, y_{22}, y_{33}, y_{123}, y_{12}, y_{21}, y_{13}, y_{231}, y_{23}, y_{32}, y_{31}, y_{312} =$ **variables** 

- $\alpha_1, \alpha_2, \alpha_3 =$  ternary constants of algebraic equations
  - $\gamma$  = liquid phase activity coefficient
  - $\pi = \text{total pressure}$

#### Subscripts

1, 2, 3, i, j = components

m = mixture

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